

# Blind Adaptive Algorithms for Decision Feedback DS-CDMA Receivers in Multipath Channels

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**Abstract**—In this work we examine blind adaptive and iterative decision feedback (DF) receivers for direct sequence code division multiple access (DS-CDMA) systems in frequency selective channels. Code-constrained minimum variance (CMV) and constant modulus (CCM) design criteria for DF receivers based on constrained optimization techniques are investigated for scenarios subject to multipath. Computationally efficient blind adaptive stochastic gradient (SG) and recursive least squares (RLS) algorithms are developed for estimating the parameters of DF detectors along with successive, parallel and iterative DF structures. A novel successive parallel arbitrated DF scheme is presented and combined with iterative techniques for use with cascaded DF stages in order to mitigate the deleterious effects of error propagation. Simulation results for an uplink scenario assess the algorithms, the blind adaptive DF detectors against linear receivers and evaluate the effects of error propagation of the new cancellations techniques against previously reported approaches. In this work we examine blind adaptive and iterative decision feedback (DF) receivers for direct sequence code division multiple access (DS-CDMA) systems in frequency selective channels. Code-constrained minimum variance (CMV) and constant modulus (CCM) design criteria for DF receivers based on constrained optimization techniques are investigated for scenarios subject to multipath. Computationally efficient blind adaptive stochastic gradient (SG) and recursive least squares (RLS) algorithms are developed for estimating the parameters of DF detectors along with successive, parallel and iterative DF structures. A novel successive parallel arbitrated DF scheme is presented and combined with iterative techniques for use with cascaded DF stages in order to mitigate the deleterious effects of error propagation. Simulation results for an uplink scenario assess the algorithms, the blind adaptive DF detectors against linear receivers and evaluate the effects of error propagation of the new cancellations techniques against previously reported approaches.

## I. INTRODUCTION

CODE division multiple access (CDMA) implemented with direct sequence (DS) spread-spectrum signalling is amongst the most promising multiple access technologies for current and future communication systems. Such services include third-generation cellular telephony, indoor wireless networks, terrestrial and satellite communication systems. The advantages of CDMA include good performance in multi-path channels, flexibility in the allocation of channels, increased capacity in bursty and fading environments and the ability

to share bandwidth with narrowband communication systems without deterioration of either's systems performance [1], [2].

Demodulating a desired user in a DS-CDMA network requires processing the received signal in order to mitigate different types of interference, namely, narrowband interference (NBI), multi-access interference (MAI), inter-symbol interference (ISI) and the noise at the receiver. The major source of interference in most CDMA systems is MAI, which arises due to the fact that users communicate through the same physical channel with non-orthogonal signals. The conventional (single-user) receiver that employs a filter matched to the signature sequence does not suppress MAI and is very sensitive to differences in power between the received signals (near-far problem). Multiuser detection has been proposed as a means to suppress MAI, increasing the capacity and the performance of CDMA systems [1], [2]. The optimal multiuser detector of Verdu [3] suffers from exponential complexity and requires the knowledge of timing, amplitude and signature sequences. This fact has motivated the development of various sub-optimal strategies: the linear [4] and decision feedback [5] receivers, the successive interference canceller [6] and the multistage detector [7]. For uplink scenarios, decision feedback detection, which is relatively simple and performs linear interference suppression followed by interference cancellation was shown to provide substantial gains over linear detection [5], [8], [10], [11].

When used with short code or repeated spreading codes, adaptive signal processing methods are suitable to CDMA systems because they can track the highly dynamic conditions often encountered in such systems due to the mobility of mobile terminals and the random nature of the channel access. Adaptive techniques can also alleviate the computational complexity required for parameter estimation. In particular, blind adaptive signal processing is an interesting alternative for situations where a receiver loses track of the desired user and/or a training sequence is not available. In this context, blind linear receivers for DS-CDMA have been proposed in the last years to suppress MAI [13], [14], [15], [16], [17], [18]. Blind linear solutions for flat channels have been reported for the first time in [13], where the blind detector was designed on the basis of the minimum output energy (MOE) or minimum variance (MV). Following the initial success of the MV receiver [13], blind receivers using the constant modulus (CM) criterion, which outperformed their MV counterparts, were reported in [14], [16] and [17]. In this context, the work by Tugnait and Li [17] is an inverse filtering criterion and does not exploit the energy contained in the signal copies available in multipath, leading to performance degradation as compared

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to supervised solutions. In order to improve performance and close the gap between blind and trained solutions, Xu and Tsatsanis [15] exploited the multipath components through a constrained MV (CMV) method [15] that treats different signal copies as variables and jointly optimizes the receiver and channel parameters. Another solution that outperforms the CMV technique of [15] was proposed by Xu and Liu [18] for multipath environments, in which constrained adaptive linear receivers are derived based upon the joint optimization of channel and receiver parameters in accordance with the constant modulus criterion. Recently, a code-constrained CM design for linear receivers and an RLS algorithm, that outperform previous approaches, were presented in [21] for a downlink scenario.

Although relatively simple, DF structures can perform significantly better than linear systems and the existing work on blind adaptive DF receivers was restricted to single-path channels solutions [22], [23], [24] and have to be modified for multipath. Detectors with DF are especially interesting because they offer the possibility of different types of cancellation, namely, successive [8], [9], parallel [10] and iterative [11], [12], which lead to different performances and degrees of robustness against error propagation. This paper addresses blind adaptive DF detection for multipath channels in DS-CDMA systems based on constrained optimization techniques using the MV and CM criteria. The CMV and CCM solutions for the design of blind DF CDMA receivers are presented and then computationally efficient blind adaptive algorithms are developed for MAI, intersymbol interference (ISI) suppression and channel estimation. The second contribution of this work is a novel successive parallel arbitrated DF structure based on the recent concept of parallel arbitration [25]. The new DF detector is then combined with iterative cascaded DF stages, resulting in an improved DF receiver structure that is compared with previously reported methods. Computer simulations experiments show the effectiveness of the proposed blind DF system for refining soft estimates and mitigating the effects of error propagation.

This paper is organized as follows. Section II briefly describes the DS-CDMA communication system model. The constrained decision feedback receivers and the blind channel estimation procedure are described in Section III. Section IV is devoted to the successive parallel arbitrated and iterative DF cancellation techniques, whereas Section V is dedicated to the derivation of adaptive SG algorithms and RLS type algorithms. Section VI presents and discusses the simulation results and Section VII gives the conclusions of this work.

## II. DS-CDMA SYSTEM MODEL

Let us consider the uplink of a symbol synchronous binary phase-shift keying (BPSK) DS-CDMA system with  $K$  users,  $N$  chips per symbol and  $L_p$  propagation paths. It should be remarked that a synchronous model is assumed for simplicity, although it captures most of the features of more realistic asynchronous models with small to moderate delay spreads. The baseband signal transmitted by the  $k$ -th active user to the

base station is given by

$$x_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k(i) s_k(t - iT) \quad (1)$$

where  $b_k(i) \in \{\pm 1\}$  denotes the  $i$ -th symbol for user  $k$ , the real valued spreading waveform and the amplitude associated with user  $k$  are  $s_k(t)$  and  $A_k$ , respectively. The spreading waveforms are expressed by  $s_k(t) = \sum_{i=1}^N a_k(i) \phi(t - iT_c)$ , where  $a_k(i) \in \{\pm 1/\sqrt{N}\}$ ,  $\phi(t)$  is the chip waveform,  $T_c$  is the chip duration and  $N = T/T_c$  is the processing gain. Assuming that the receiver is synchronised with the main path, the coherently demodulated composite received signal is

$$r(t) = \sum_{k=1}^K \sum_{l=0}^{L_p-1} h_{k,l}(t) x_k(t - \tau_{k,l}) + n(t) \quad (2)$$

where  $h_{k,l}(t)$  and  $\tau_{k,l}$  are, respectively, the channel coefficient and the delay associated with the  $l$ -th path and the  $k$ -th user. Assuming that  $\tau_{k,l} = lT_c$ , the channel is constant during each symbol interval and the spreading codes are repeated from symbol to symbol, the received signal  $r(t)$  after filtering by a chip-pulse matched filter and sampled at chip rate yields the  $M$ -dimensional received vector

$$\mathbf{r}(i) = \sum_{k=1}^K \mathbf{H}_k(i) A_k \mathbf{S}_k \mathbf{b}_k(i) + \mathbf{n}(i) \quad (3)$$

where  $M = N + L_p - 1$ ,  $\mathbf{n}(i) = [n_1(i) \dots n_M(i)]^T$  is the complex Gaussian noise vector with  $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2 \mathbf{I}$ , where  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and Hermitian transpose, respectively,  $E[\cdot]$  stands for ensemble average, the user symbol vector is  $\mathbf{b}_k(i) = [b_k(i) \dots b_k(i - L_s + 1)]^T$ , the amplitude of user  $k$  is  $A_k$ , the channel vector of user  $k$  is  $\mathbf{h}_k(i) = [h_{k,0}(i) \dots h_{k,L_p-1}(i)]^T$ ,  $L_s$  is the ISI span and the  $(L_s \times N) \times L_s$  diagonal matrix  $\mathbf{S}_k$  with  $N$ -chips shifted versions of the signature of user  $k$  is given by

$$\mathbf{S}_k = \begin{bmatrix} \mathbf{s}_k & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{s}_k & \ddots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{s}_k \end{bmatrix} \quad (4)$$

where  $\mathbf{s}_k = [a_k(1) \dots a_k(N)]^T$  is the signature sequence for the  $k$ -th user and the  $M \times (L_s \times N)$  channel matrix  $\mathbf{H}_k(i)$  for user  $k$  is

$$\mathbf{H}_k(i) = \begin{bmatrix} h_{k,0}(i) & \dots & h_{k,L_p-1}(i) & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & h_{k,0}(i) & \dots & h_{k,L_p-1}(i) \end{bmatrix} \quad (5)$$

where  $h_{k,l}(i) = h_{k,l}(iT_c)$ . The MAI comes from the non-orthogonality between the received signature sequences, whereas the ISI span  $L_s$  depends on the length of the channel response, which is related to the length of the chip sequence. For  $L_p = 1$ ,  $L_s = 1$  (no ISI), for  $1 < L_p \leq N$ ,  $L_s = 2$ , for  $N < L_p \leq 2N$ ,  $L_s = 3$ .

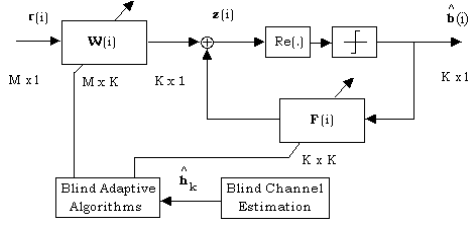


Fig. 1. Block diagram of a blind multiuser decision feedback receiver.

### III. BLIND DECISION FEEDBACK CONSTRAINED RECEIVERS

Let us describe the design of synchronous blind decision feedback constrained detectors, as the one shown in Fig. 1. It should be remarked that portions of the material presented here were presented in [19]. Consider the received vector  $\mathbf{r}(i)$ , and let us introduce the  $M \times L_p$  constraint matrix  $\mathbf{C}_k$  that contains one-chip shifted versions of the signature sequence for user  $k$ :

$$\mathbf{C}_k = \begin{bmatrix} a_k(1) & \mathbf{0} & & \\ \vdots & \ddots & a_k(1) & \\ a_k(N) & \vdots & & \\ \mathbf{0} & \ddots & a_k(N) & \end{bmatrix} \quad (6)$$

The input to the hard decision device, depicted in Fig. 1, corresponding to the  $i$ th symbol is

$$\mathbf{z}(i) = \mathbf{W}^H(i)\mathbf{r}(i) - \mathbf{F}^H(i)\hat{\mathbf{b}}(i) \quad (7)$$

where the input  $\mathbf{z}(i) = [z_1(i) \dots z_K(i)]^T$ ,  $\mathbf{W}(i) = [\mathbf{w}_1 \dots \mathbf{w}_K]$  is  $M \times K$  the feedforward matrix,  $\hat{\mathbf{b}}(i) = [b_1(i) \dots b_K(i)]^T$  is the  $K \times 1$  vector of estimated symbols, which are fed back through the  $K \times K$  feedback matrix  $\mathbf{F}(i) = [\mathbf{f}_1(i) \dots \mathbf{f}_K(i)]$ . Generally, the DF receiver design is equivalent to determining for user  $k$  a feedforward filter  $\mathbf{w}_k(i)$  with  $M$  elements and a feedback one  $\mathbf{f}_k(i)$  with  $K$  elements that provide an estimate of the desired symbol:

$$z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i), \quad k = 1, 2, \dots, K \quad (8)$$

where  $\hat{\mathbf{b}}(i) = \text{sgn}[\Re(\mathbf{W}^H(i)\mathbf{r}(i))]$  is the vector with initial decisions provided by the linear section,  $\mathbf{w}_k$  and  $\mathbf{f}_k$  are optimized by the MV or the CM cost functions, subject to a set multipath constraints given by  $\mathbf{C}_k^H \mathbf{w}_k(i) = \mathbf{h}_k(i)$  for the MV case, or  $\mathbf{C}_k^H \mathbf{w}_k(i) = \nu \mathbf{h}_k(i)$  for the CM case, where  $\nu$  is a constant to ensure the convexity of the CM-based receiver and  $\mathbf{h}_k(i)$  is the  $k$ th user channel vector. In particular, the feedback filter  $\mathbf{f}_k(i)$  of user  $k$  has a number of non-zero coefficients corresponding to the available number of feedback connections for each type of cancellation structure. The final detected symbol is obtained with:

$$\hat{b}_k^f(i) = \text{sgn}(\Re[z_k(i)]) = \text{sgn}(\Re[\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)]), \quad (9)$$

where  $\Re(\cdot)$  selects the real part and  $\text{sgn}(\cdot)$  is the signum function. For successive DF (S-DF) [8], the  $K \times K$  matrix

$\mathbf{F}(i)$  is strictly lower triangular, whereas for parallel DF (P-DF) [10], [11]  $\mathbf{F}(i)$  is full and constrained to have zeros on the main diagonal in order to avoid cancelling the desired symbols. The S-DF structure is optimal in the sense of that it achieves the sum capacity of the synchronous CDMA channel with AWGN [9]. In addition, the S-DF scheme is less affected by error propagation although it generally does not provide uniform performance over the user population, which is a desirable characteristic for uplink scenarios. In this context, the P-DF system can offer uniform performance over the users but it suffers from error propagation. In order to design the DF receivers and satisfy the constraints of S-DF and P-DF structures, the designer must obtain the vector with initial decisions  $\hat{\mathbf{b}}(i) = \text{sgn}[\Re(\mathbf{W}^H(i)\mathbf{r}(i))]$  and then resort to the following cancellation approach. The non-zero part of the filter  $\mathbf{f}_k$  corresponds to the number of used feedback connections and to the users to be cancelled. For the S-DF, the number of feedback elements and their associated number of non-zero filter coefficients in  $\mathbf{f}_k$  (where  $k$  goes from the second detected user to the last one) range from 1 to  $K - 1$ . For the P-DF, the feedback connections used and their associated number of non-zero filter coefficients in  $\mathbf{f}_k$  are equal to  $K - 1$  for all users and the matrix  $\mathbf{F}(i)$  has zeros on the main diagonal to avoid cancelling the desired symbols.

In what follows, constrained CM and MV design criteria for DF detectors are presented. The CMV design for DF receivers generalizes the work on linear structures of Xu and Tsatsanis [15], whereas the CCM design is proposed here for both linear and DF schemes.

#### A. DF Constrained Constant Modulus (DF-CCM) Receivers

To describe the DF-CCM receiver design let us consider the CM cost function:

$$J_{CM}(i) = E[(|\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)|^2 - 1)^2] = E[(|z_k(i)|^2 - 1)^2] \quad (10)$$

subject to  $\mathbf{C}_k^H \mathbf{w}_k(i) = \nu \mathbf{h}_k(i)$ , where  $z_k(i) = \mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)$ . Assuming that the channel vector  $\mathbf{h}_k$  is known, let us consider the unconstrained cost function  $J'_{CM}(i) = E[(|\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)|^2 - 1)^2] + 2\Re[(\mathbf{C}_k^H \mathbf{w}_k(i) - \nu \mathbf{h}_k(i))^H \boldsymbol{\lambda}]$ , where  $\boldsymbol{\lambda}$  is a vector of complex Lagrange multipliers. The function  $J'_{CM}(i)$  is minimized with respect to  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$  under the set of constraints  $\mathbf{C}_k^H \mathbf{w}_k(i) = \nu \mathbf{h}_k(i)$ . Taking the gradient terms of  $J_{CM}(i)'$  with respect to  $\mathbf{w}_k(i)$  and setting them to zero we have  $\nabla J_{CM}(i)' = 2E[(|\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)|^2 - 1)\mathbf{r}(i)(\mathbf{r}^H(i)\mathbf{w}_k(i) - \hat{\mathbf{b}}^H(i)\mathbf{f}_k(i))] + 2\mathbf{C}_k\boldsymbol{\lambda} = \mathbf{0}$ , then rearranging the terms we obtain  $E[|z_k(i)|^2\mathbf{r}(i)\mathbf{r}^H(i)]\mathbf{w}_k(i) = E[z_k^*(i)\mathbf{r}(i)] + E[|z_k(i)|^2\mathbf{r}(i)\hat{\mathbf{b}}^H(i)]\mathbf{f}_k(i) - \mathbf{C}_k\boldsymbol{\lambda}$  and consequently  $\mathbf{w}_k(i) = \mathbf{R}_k^{-1}(i)[\mathbf{d}_k(i) + \mathbf{T}_k(i)\mathbf{f}_k(i) - \mathbf{C}_k\boldsymbol{\lambda}]$ , where  $\mathbf{R}_k(i) = E[|z_k(i)|^2\mathbf{r}(i)\mathbf{r}^H(i)]$ ,  $\mathbf{T}_k(i) = E[|z_k(i)|^2\mathbf{r}(i)\hat{\mathbf{b}}^H(i)]$ ,  $\mathbf{d}_k(i) = E[z_k^*(i)\mathbf{r}(i)]$  and the asterisk denotes complex conjugation. Using the constraint  $\mathbf{C}_k^H \mathbf{w}_k(i) = \nu \mathbf{h}_k(i)$  we arrive at the expression for the Lagrange multiplier  $\boldsymbol{\lambda} = (\mathbf{C}_k^H \mathbf{R}_k^{-1}(i)\mathbf{C}_k)^{-1}(\mathbf{C}_k^H \mathbf{R}_k^{-1}(i)\mathbf{T}_k(i)\mathbf{f}_k(i) + \mathbf{C}_k^H \mathbf{R}_k^{-1}(i)\mathbf{d}_k(i) - \nu \mathbf{h}_k(i))$ . By substituting  $\boldsymbol{\lambda}$  into  $\mathbf{w}_k(i) = \mathbf{R}_k^{-1}(i)[\mathbf{d}_k(i) + \mathbf{T}_k(i)\mathbf{f}_k(i) - \mathbf{C}_k\boldsymbol{\lambda}]$  we obtain the solution for

the feedforward section of the DF-CCM receiver:

$$\mathbf{w}_k(i) = \mathbf{R}_k^{-1}(i) \left[ \mathbf{d}_k(i) + \mathbf{T}_k(i) \mathbf{f}_k(i) - \mathbf{C}_k (\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k)^{-1} \times \left( \mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{T}_k(i) \mathbf{f}_k(i) + \mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{d}_k(i) - \nu \mathbf{h}_k(i) \right) \right] \quad (11)$$

where the expression in (11) is a function of previous values of  $\mathbf{w}_k(i)$  and the channel  $\mathbf{h}_k(i)$ . To obtain the CCM solution for the parameter vector  $\mathbf{f}_k$  of the feedback section, we compute the gradient terms of  $J'_{CM}$  with respect to  $\mathbf{f}_k$  and by setting them to zero we have  $\nabla J'_{CM}(i) = 2E[(|z_k(i)|^2 - 1)\hat{\mathbf{b}}(i)(\mathbf{r}^H(i)\mathbf{w}_k(i) - \hat{\mathbf{b}}^H(i)\mathbf{f}_k(i))] = \mathbf{0}$ , then rearranging the terms we get  $E[|z_k(i)|^2\hat{\mathbf{b}}(i)\hat{\mathbf{b}}^H(i)]\mathbf{f}_k(i) = E[|z_k(i)|^2\hat{\mathbf{b}}(i)\mathbf{r}^H(i)]\mathbf{w}_k(i) - E[z_k^*(i)\hat{\mathbf{b}}(i)]$  and consequently we have

$$\mathbf{f}_k(i) = \mathbf{I}_k^{-1} [\mathbf{T}_k^H(i)\mathbf{w}_k(i) - \mathbf{v}_k(i)] \quad (12)$$

where  $\mathbf{I}_k = E[|z_k(i)|^2\hat{\mathbf{b}}(i)\hat{\mathbf{b}}^H(i)]$  and  $\mathbf{v}_k = E[z_k^*(i)\hat{\mathbf{b}}(i)]$ . We remark that (11) and (12) should be iterated in order to estimate the desired user symbols. The CCM linear receiver solution proposed in [21] is obtained by making  $\mathbf{f}_k(i) = \mathbf{0}$  in (11). An analysis of the CCM method in the Appendix I examines its convergence properties for the linear receiver case, extending previous results on its convexity for both complex and multipath signals. Since the optimization of the CCM cost function for a linear receiver ( $\mathbf{f}_k(i) = \mathbf{0}$ ) is a convex optimization, as shown in the Appendix I, it provides a good starting point for performing the cancellation of the associated users by the feedforward section of the DF-CCM receiver.

#### B. DF Constrained Minimum Variance (DF-CMV) Receivers

The DF-CMV receiver design resembles the DF-CCM design and considers the following cost function :

$$J_{MV} = E[|\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)|^2] \quad (13)$$

subject to  $\mathbf{C}_k^H \mathbf{w}_k(i) = \mathbf{h}_k(i)$ . Given the channel vector  $\mathbf{h}_k(i)$ , let us consider the unconstrained cost function  $J'_{MV}(i) = E[|\mathbf{w}_k^H(i)\mathbf{r}(i) - \mathbf{f}_k^H(i)\hat{\mathbf{b}}(i)|^2] + 2\Re[(\mathbf{C}_k^H \mathbf{w}_k(i) - \mathbf{h}_k(i))^H \boldsymbol{\lambda}]$ , where  $\boldsymbol{\lambda}$  is a vector of complex Lagrange multipliers, and minimize  $J'_{MV}(i)$  with respect to  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$  under the set of constraints  $\mathbf{C}_k^H \mathbf{w}_k(i) = \mathbf{h}_k(i)$ . By taking the gradient terms of  $J'_{MV}(i)$  with respect to  $\mathbf{w}_k(i)$  and setting them to zero we have  $\nabla J'_{MV}(i) = E[\mathbf{r}(i)(\mathbf{r}^H(i)\mathbf{w}_k(i) - \hat{\mathbf{b}}^H(i)\mathbf{f}_k(i))] + 2\mathbf{C}_k\boldsymbol{\lambda} = \mathbf{0}$ , then rearranging the terms we obtain  $E[\mathbf{r}(i)\mathbf{r}^H(i)]\mathbf{w}_k(i) = E[\mathbf{r}(i)\hat{\mathbf{b}}^H(i)]\mathbf{f}_k(i) - 2\mathbf{C}_k\boldsymbol{\lambda}$  and consequently  $\mathbf{w}_k(i) = \mathbf{R}^{-1}(i)[\mathbf{T}(i)\mathbf{f}_k(i) - 2\mathbf{C}_k\boldsymbol{\lambda}]$ , where the covariance matrix is  $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$  and  $\mathbf{T}(i) = E[\mathbf{r}(i)\hat{\mathbf{b}}(i)]$ . Using the constraint  $\mathbf{C}_k^H \mathbf{w}_k(i) = \mathbf{h}_k(i)$  we arrive at the expression for the Lagrange multiplier  $\boldsymbol{\lambda} = (\mathbf{C}_k^H \mathbf{R}^{-1}(i) \mathbf{C}_k)^{-1} (\mathbf{C}_k^H \mathbf{R}^{-1}(i) \mathbf{T}(i) \mathbf{f}_k(i) - \mathbf{h}_k(i))/2$ . By substituting  $\boldsymbol{\lambda}$  into  $\mathbf{w}_k(i) = \mathbf{R}^{-1}(i)[\mathbf{T}(i)\mathbf{f}_k(i) - 2\mathbf{C}_k\boldsymbol{\lambda}]$  we obtain the solution for the feedforward section of the DF-CMV receiver:

$$\mathbf{w}_k(i) = \mathbf{R}^{-1}(i) \left[ \mathbf{T}(i)\mathbf{f}_k(i) - \mathbf{C}_k (\mathbf{C}_k^H \mathbf{R}^{-1}(i) \mathbf{C}_k)^{-1} \times \right.$$

$$\left. (\mathbf{C}_k^H \mathbf{R}^{-1}(i) \mathbf{T}(i) \mathbf{f}_k(i) - \mathbf{h}_k(i)) \right] \quad (14)$$

Next, we compute the gradient terms of  $J'_{MV}(i)$  with respect to  $\mathbf{f}_k(i)$  and set them to zero to get  $\nabla J'_{MV}(i) = E[\hat{\mathbf{b}}(i)(\mathbf{r}^H(i)\mathbf{w}_k(i) - \hat{\mathbf{b}}^H(i)\mathbf{f}_k(i))] = \mathbf{0}$ , then rearranging the terms we have  $E[\hat{\mathbf{b}}(i)\hat{\mathbf{b}}^H(i)]\mathbf{f}_k(i) = E[\hat{\mathbf{b}}(i)\mathbf{r}^H(i)]\mathbf{w}_k(i)$  and consequently we obtain

$$\mathbf{f}_k(i) = \mathbf{B}^{-1}(i) [\mathbf{T}^H(i)\mathbf{w}_k(i)] \quad (15)$$

where  $\mathbf{B}(i) = E[\hat{\mathbf{b}}(i)\hat{\mathbf{b}}^H(i)]$ . At this point, the designer can avoid the inversion of  $\mathbf{B}(i)$  by using a judicious approximation, that is  $\mathbf{I} \approx E[\hat{\mathbf{b}}(i)\hat{\mathbf{b}}^H(i)]$  [2], which is verified unless the error rate is high. Hence, the feedback section filter can be designed as given by  $\mathbf{f}_k(i) \approx \mathbf{T}^H(i)\mathbf{w}_k(i)$ . It should also be noted that by making  $\mathbf{f}_k(i) = \mathbf{0}$  we arrive at the solution of Xu and Tsatsanis in [15].

#### C. Blind Channel Estimation

The solutions for the CCM and CMV DF receivers assume the knowledge of the channel parameters. However, in applications where multipath is present these parameters are not known and thus channel estimation is required. To blindly estimate the channel we use the method of [15], [26]:

$$\hat{\mathbf{h}}_k(i) = \arg \min_{\mathbf{h}_k} \mathbf{h}_k^T \mathbf{C}_k^T \mathbf{R}^{-p}(i) \mathbf{C}_k \mathbf{h}_k \quad (16)$$

subject to  $\|\hat{\mathbf{h}}_k\| = 1$ , where  $p$  is an integer and whose solution is the eigenvector corresponding to the minimum eigenvalue of the  $L_p \times L_p$  matrix  $\mathbf{C}_k^T \mathbf{R}^{-p}(i) \mathbf{C}_k$ . For the CCM receiver we employ  $\mathbf{R}_k(i)$  in lieu of  $\mathbf{R}(i)$  (used for the CMV) for channel estimation. The use of  $\mathbf{R}_k(i)$  instead of  $\mathbf{R}$  avoids the estimation of both  $\mathbf{R}(i)$  and  $\mathbf{R}_k(i)$ , and shows no performance loss as verified in our studies and explained in Appendix IV. The values of  $p$  are restricted to 1 even though the performance of the channel estimator and consequently of the receiver can be improved by increasing  $p$ .

### IV. SUCCESSIVE PARALLEL ARBITRATED AND ITERATIVE DF DETECTION

In this section, we present novel iterative techniques, which are based on the recently introduced concept of parallel arbitration [25], and combine them with iterative cascaded DF stages [11], [12]. The motivation for the novel DF structures is to mitigate the effects of error propagation often found in P-DF structures [11], [12], that are of great interest for uplink scenarios due to its capability of providing uniform performance over the users. The basic idea is to improve the S-DF structure using parallel searches and then combine it with an iterative technique, where the second stage uses a P-DF system to equalize the performance of the users.

#### A. Successive Parallel Arbitrated DF Detection

The idea of parallel arbitration is to employ successive interference cancellation (SIC) to rapidly converge to a local maximum of the likelihood function and, by running parallel

branches of SIC with different orders of cancellation, one can arrive at sufficiently different local maxima [25]. In order to obtain the benefits of parallel search, the candidates should be arbitrated, yielding different estimates of a symbol. The estimate of a symbol that has the highest likelihood is then selected at the output.

Unlike the work of Barriac and Madhow [25] that employed matched filters as the starting point, we adopt blind DF receivers as the initial condition. The concept of parallel arbitration is thus incorporated into a DF detector structure, that applies linear interference suppression followed by SIC and yields improved starting points as compared to matched filters. It is also worth noting that our approach does not require regeneration as occurs with the original PASIC in [25] because the blind adaptive filters automatically compute the coefficients for interference cancellation. A block diagram of the proposed scheme, denoted successive parallel arbitrated decision feedback (SPA-DF), is shown in Fig. 2.

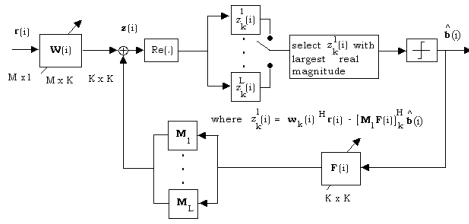


Fig. 2. Block diagram of the proposed blind SPA-DF receiver.

Following the schematics of Fig. 2, the user  $k$  output of the parallel branch  $l$  ( $l = 1, \dots, L$ ) for the SPA-DF receiver structure is given by:

$$z_k^l(i) = \mathbf{w}_k^H(i) \mathbf{r}(i) - [\mathbf{M}_l \mathbf{F}]_k^H \hat{\mathbf{b}}(i) \quad (17)$$

where the vector with initial decisions is  $\hat{\mathbf{b}}(i) = \text{sgn}[\Re(\mathbf{W}^H(i) \mathbf{r}(i))]$  and the matrices  $\mathbf{M}_l$  are permuted square identity ( $\mathbf{I}_K$ ) matrices with dimension  $K$  whose structures for an  $L = 4$ -branch SPA-DF scheme are given by:

$$\mathbf{M}_1 = \mathbf{I}_K, \quad \mathbf{M}_2 = \begin{bmatrix} \mathbf{0}_{K/4, 3K/4} & \mathbf{I}_{3K/4} \\ \mathbf{I}_{K/4} & \mathbf{0}_{K/4, 3K/4} \end{bmatrix}, \quad \mathbf{M}_3 = \begin{bmatrix} \mathbf{0}_{K/2} & \mathbf{I}_{K/2} \\ \mathbf{I}_{K/2} & \mathbf{0}_{K/2} \end{bmatrix}, \quad \mathbf{M}_4 = \begin{bmatrix} 0 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 0 \end{bmatrix} \quad (18)$$

where  $\mathbf{0}_{m,n}$  denotes an  $m \times n$ -dimensional matrix full of zeros and the structures of the matrices  $\mathbf{M}_l$  correspond to phase shifts regarding the cancellation order of the users. Indeed, the purpose of the matrices in (18) is to change the order of cancellation. When  $\mathbf{M} = \mathbf{I}$  the order of cancellation is a simple successive cancellation (S-DF) based upon the user powers (the same as [8], [9]). Specifically, the above matrices perform the cancellation with the following order with respect to user powers:  $\mathbf{M}_1$  with  $1, \dots, K$ ;  $\mathbf{M}_2$  with  $K/4, K/4 + 1, \dots, K, 1, \dots, K/4 - 1$ ;  $\mathbf{M}_3$  with  $K/2, K/2 + 1, \dots, K, 1, \dots, K/2 - 1$ ;  $\mathbf{M}_4$  with  $K, \dots, 1$  (reverse order). For more branches, additional phase shifts are applied with

respect to user cancellation ordering. It is also worth noting that different update orders have been tried although they did not result in performance improvements. For the proposed SPA-DF, the number of feedback elements used and their associated number of non-zero filter coefficients in  $\mathbf{f}_k$  (where  $k$  goes from the second detected user to the last one) range from 1 to  $K - 1$  according to the branch  $l$  and the matrix  $\mathbf{M}_l$ .

The final output  $\hat{b}_k^f(i)$  of the SPA-DF detector chooses the estimate of the  $L$  candidates as described by:

$$\hat{b}_k^f(i) = \text{sgn} \left[ \arg \max_{1 \leq l \leq L} |\Re(z_k^l(i))| \right] \quad (19)$$

where the selected estimate is the one with largest real magnitude, that forms the vector of final decisions  $\hat{\mathbf{b}}_k^f(i) = [\hat{b}_1^f(i) \dots \hat{b}_K^f(i)]^T$ . The number of parallel branches  $L$  that yield detection candidates is a parameter that must be chosen by the designer. Our studies and computer simulations indicate that  $L = 4$  achieves most of the gains of the proposed structure and offers a good trade-off between performance and complexity. In terms of complexity the SPA-DF system employs the same filters, namely  $\mathbf{W}(i)$  and  $\mathbf{F}(i)$ , of the traditional S-DF and requires additional arithmetic operations to compute the parallel arbitrated candidates. As occurs with S-DF receivers, a disadvantage of the SPA-DF detector is that it generally does not provide uniform performance over the user population. Specifically, in a scenario with tight power control successive techniques tend to favor the last detected users, resulting in non-uniform performance. To equalize the performance of the users an iterative technique with multiple stages can be used.

### B. Iterative Successive Parallel Arbitrated DF Detection

In [11], Woodward *et al.* presented an iterative detector with an S-DF in the first stage and P-DF or S-DF structures, with users being demodulated in reverse order, in the second stage. The work of [11] was then extended to account for coded systems and training-based reduced-rank filters [12]. Differently from [11], [12], we focus on blind adaptive receivers, uncoded systems and combine the proposed SPA-DF structure with iterative detection. An iterative receiver with hard-decision feedback is defined by the recursion:

$$\mathbf{z}^{(m+1)}(i) = \mathbf{W}^H(i) \mathbf{r}(i) - \mathbf{F}^H(i) \hat{\mathbf{b}}^{(m)}(i) \quad (20)$$

where the filters  $\mathbf{W}$  and  $\mathbf{F}$  can be S-DF or P-DF structures, and  $\hat{\mathbf{b}}^{(m)}(i)$  is the vector of tentative decisions from the preceding iteration, where we have:

$$\hat{\mathbf{b}}^{(1)}(i) = \text{sgn} \left( \Re \left[ \mathbf{W}^H(i) \mathbf{r}(i) \right] \right) \quad (21)$$

$$\hat{\mathbf{b}}^{(m)}(i) = \text{sgn} \left( \Re \left[ \mathbf{z}^{(m)}(i) \right] \right), \quad m > 1 \quad (22)$$

where the number of stages  $m$  depends on the application. Additional stages can be added where the order of the users is reversed from stage to stage.

To equalize the performance over the user population, we consider the two-stage structure shown in Fig. 3. The first stage is an SPA-DF scheme with filters  $\mathbf{W}^1$  and  $\mathbf{F}^1$ . The tentative

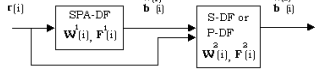


Fig. 3. Block diagram of the two-stage DF receiver with SPA-DF scheme in the first stage. The second stage can employ S-DF or P-DF structures to demodulate users in reverse order relative to the first branch of the first stage, that uses S-DF detection.

decisions are passed to the second stage, which consists of an S-DF or an P-DF detector with filters  $\mathbf{W}^2$  and  $\mathbf{F}^2$ . The users in the second stage are demodulated successively and in reverse order relative to the first branch of the SPA-DF structure (a conventional S-DF). The resulting iterative receiver system is denoted ISPAS-DF when an S-DF scheme is deployed in the second stage, whereas for a P-DF filters in the second stage the overall scheme is called ISPAP-DF. The output of the second stage of the resulting scheme is expressed by:

$$z_j^{(2)}(i) = [\mathbf{M}\mathbf{W}^2(i)]_j^H \mathbf{r}(i) - [\mathbf{M}\mathbf{F}^2(i)]_j^H \hat{\mathbf{b}}^{(2)}(i) \quad (23)$$

where  $z_j$  is the  $j$ th component of the soft output vector  $\mathbf{z}$ ,  $\mathbf{M}$  is a square permutation matrix with ones along the reverse diagonal and zeros elsewhere (similar to  $\mathbf{M}_4$  in (18)),  $[\cdot]_j$  denotes the  $j$ th column of the argument (a matrix), and  $b_j^m(i) = \text{sgn}[\Re(z_j^m(i))]$ . Note that additional stages can be included or the SPA-DF scheme can be used in the second stage, even though our studies indicate that the gains in performance are marginal. Hence, the two-stage structure is adopted for the rest of this work. It should also be remarked that, due to the difficulty of theoretically analyzing parallel arbitrated and iterative schemes, our analysis in Section VI is mainly focused on computer simulation experiments. A theoretical analysis of iterative DF schemes constitutes an open topic which is beyond the scope of this paper.

## V. ADAPTIVE ALGORITHMS

In this section we describe stochastic gradient (SG) and recursive least squares (RLS) algorithms for the blind estimation of the channel, the feedforward and feedback sections of DF receivers using the CM and MV criteria along with constrained optimization techniques, as illustrated in Fig. 1. The CMV-based algorithms are extensions for DF detection of the techniques proposed by Xu and Tsatsanis in [15]. The CCM-SG recursions represent an extension of the work of [18] for complex signals and DF receivers, whereas the CCM-RLS algorithms are novel for both linear and DF structures.

It should be emphasized that the SG solutions presented in this section differ from those reported in a previous work [20] in the sense that the blind channel estimation is decoupled from the feedforward and feedback recursions. Indeed, we adopt the SG blind channel estimation reported in [27] that has been shown to outperform the one proposed in [15]. Our studies also reveal that when the system deals with high loads ( $K$  is large) and the performance is poorer, a decoupled SG blind channel estimator, such as [27], is significantly less affected than the approach that optimizes  $\mathbf{w}_k$ ,  $\mathbf{f}_k$  and  $\hat{\mathbf{h}}_k$  as in [20]. In addition, the deployment of the SG blind estimator

of [27] with SG CCM-based algorithms considerably improves its performance, because blind channel estimators that rely on the CM criterion show poor performance and depend on other methods for initialization, as pointed out in [18].

In terms of performance, RLS recursions have the potential to achieve good performance independently of the spread of the eigenvalues of the input signal autocorrelation matrix, have faster convergence performance, show superior performance under fast frequency selective fading channels and can cope with larger systems [29] than SG techniques.

In terms of complexity SG algorithms require a number of operations that grows linearly with  $M$  and additional users in order to suppress MAI, ISI and estimate the channel [27], whereas RLS techniques have quadratic complexity implementation for MAI, ISI suppression and channel estimation.

### A. Stochastic Gradient and RLS Blind Channel Estimation

The channel estimate  $\hat{\mathbf{h}}_k(i)$  is obtained through the power method and the SG and RLS techniques described in [26]. The methods are SG and RLS adaptive version of the blind channel estimation algorithms described in (16) and introduced in [27]. The SG recursion requires only  $O(L_p^2)$  arithmetic operations to estimate the channel, against  $O(L_p^3)$  of its SVD version. For the RLS version, the SVD on the  $L_p \times L_p$  matrix  $\mathbf{C}_k^H \mathbf{R}^{-1}(i) \mathbf{C}_k$ , as stated in (16) and that requires  $O(L_p^3)$ , is avoided and replaced by a single matrix-vector multiplication, resulting in the reduction of the corresponding computational complexity on one order of magnitude and no performance loss. For the CCM-RLS algorithms,  $\mathbf{R}_k$  can be employed instead of  $\mathbf{R}$  (used for the CMV) for channel estimation to avoid the estimation of both  $\mathbf{R}$  and  $\mathbf{R}_k$ . The use of  $\mathbf{R}_k$  instead of  $\mathbf{R}$  shows no performance loss as verified in our studies and is explained in Appendix IV.

### B. Constrained Constant Modulus Stochastic Gradient (CCM-SG) Algorithm

An SG solution to (10) and (11) can be devised by using instantaneous estimates and taking the gradient terms with respect to  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$  which should adaptively minimize  $J_{CM}$  with respect to  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$ . The recursions of [27] are used to obtain channel estimates. If we consider the set of constraints  $\mathbf{C}_k^H \mathbf{w}_k(i) = \hat{\mathbf{h}}_k(i)$ , we arrive at the update equations for the estimation of  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$ :

$$\mathbf{w}_k(i+1) = \mathbf{P}_k(\mathbf{w}_k(i) - \mu_w e_k(i) z_k^*(i) \mathbf{r}(i)) + \nu \mathbf{C}_k (\mathbf{C}_k^H \mathbf{C}_k)^{-1} \hat{\mathbf{h}}_k(i) \quad (24)$$

$$\mathbf{f}_k(i+1) = \mathbf{f}_k(i) - \mu_f e_k(i) z_k^*(i) \hat{\mathbf{b}}(i) \quad (25)$$

where  $z_k(i) = \mathbf{w}_k^H(i) \mathbf{r}(i) - \mathbf{f}_k^H(i) \hat{\mathbf{b}}(i)$ ,  $e_k(i) = (|z_k(i)|^2 - 1)$  and  $\mathbf{P}_k = \mathbf{I} - \mathbf{C}_k (\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{C}_k^H$  is a matrix that projects the receiver's parameters onto another hyperplane in order to ensure the constraints.

It is worth noting that, for stability and to facilitate tuning of parameters, it is useful to employ normalized step sizes when operating in a changing environment. A normalized version of this algorithm can be devised by substituting (24) and (25) into

the CM cost function, differentiating the cost function with respect to  $\mu_w$  and  $\mu_f$ , setting it to zero and solving the new equations, as detailed in Appendix II. Hence, the normalized CCM-SG algorithm proposed here adopts variable step size mechanisms described by  $\mu_w = \frac{\mu_{0w}(|z_k(i)| - \mu_f|z_k(i)|e_k(i)\mathbf{b}^H(i)\mathbf{b}(i+1))}{|z_k(i)|e_k(i)\mathbf{r}^H(i)\mathbf{Pr}(i)}$  and  $\mu_f = \frac{\mu_{0f}(|z_k(i)| - \mu_w|z_k(i)|e_k(i)\mathbf{r}^H(i)\mathbf{Pr}(i+1))}{|z_k(i)|e_k(i)\mathbf{b}^H(i)\mathbf{b}(i)}$  where  $\mu_{0w}$  and  $\mu_{0f}$  are the convergence factors for  $\mathbf{w}_k$  and  $\mathbf{f}_k$ , respectively.

### C. Constrained Minimum Variance Stochastic Gradient (CMV-SG) Algorithm

An SG solution to (13) and (14) can be developed in an analogous form to the previous section by taking the gradient terms with respect to  $\mathbf{w}_k(i)$  and  $\mathbf{f}_k(i)$ . The recursions in [27] are used again to obtain channel estimates. The update rules for the estimation of the parameters of the feedforward and feedback sections of the DF receiver are:

$$\mathbf{w}_k(i+1) = \mathbf{P}_k(\mathbf{w}_k(i) - \mu_w z_k^*(i)\mathbf{r}(i)) + \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \hat{\mathbf{h}}_k(i) \quad (26)$$

$$\mathbf{f}_k(i+1) = \mathbf{f}_k(i) - \mu_f z_k^*(i)\hat{\mathbf{b}}(i) \quad (27)$$

A normalized version of this algorithm can also be obtained by substituting (26) and (27) into the MV cost function, differentiating it with respect to  $\mu_w$  and  $\mu_f$ , setting it to zero and solving the new equations, as described in Appendix III. Hence,  $\mu_w = \frac{\mu_{0w}(1 - \mu_f \mathbf{b}^H(i)\mathbf{b}(i))}{\mathbf{r}^H(i)\mathbf{Pr}(i)}$  and  $\mu_f = \frac{\mu_{0f}(1 - \mu_w \mathbf{r}^H(i)\mathbf{Pr}(i))}{\mathbf{b}^H(i)\mathbf{b}(i)}$ .

### D. Constrained Constant Modulus RLS (CCM-RLS) Algorithm

Given the expressions for the feedforward ( $\mathbf{w}_k$ ) and feedback ( $\mathbf{f}_k$ ) sections in (11) and (12) of the blind DF receiver, we need to estimate  $\mathbf{R}_k^{-1}(i)$ ,  $\mathbf{I}_k^{-1}(i)$  and  $(\mathbf{C}_k^T \mathbf{R}_k^{-1}(i) \mathbf{C}_k)^{-1}$  recursively to reduce the computational complexity required to invert these matrices. Using the matrix inversion lemma and Kalman RLS recursions [29] we have:

$$\mathbf{G}_k(i) = \frac{\alpha^{-1} \hat{\mathbf{R}}_k^{-1}(i-1) z_k^*(i) \mathbf{r}(i)}{1 + \alpha^{-1} \mathbf{r}^H(i) z_k(i) \hat{\mathbf{R}}_k^{-1}(i-1) z_k^*(i) \mathbf{r}(i)} \quad (28)$$

$$\hat{\mathbf{R}}_k^{-1}(i) = \alpha^{-1} \hat{\mathbf{R}}_k^{-1}(i-1) - \alpha^{-1} \mathbf{G}_k(i) z_k(i) \mathbf{r}^H(i) \hat{\mathbf{R}}_k^{-1}(i-1) \quad (29)$$

and

$$\mathbf{V}(i) = \frac{\alpha^{-1} \hat{\mathbf{I}}_k^{-1}(i-1) z_k^*(i) \hat{\mathbf{b}}(i)}{1 + \alpha^{-1} \hat{\mathbf{b}}^H(i) z_k(i) \hat{\mathbf{I}}_k^{-1}(i-1) z_k^*(i) \hat{\mathbf{b}}(i)} \quad (30)$$

$$\hat{\mathbf{I}}_k^{-1}(i) = \alpha^{-1} \hat{\mathbf{I}}_k^{-1}(i-1) - \alpha^{-1} \mathbf{V}(i) z_k(i) \hat{\mathbf{b}}^H(i) \hat{\mathbf{I}}_k^{-1}(i-1) \quad (31)$$

where  $0 < \alpha < 1$  is the forgetting factor. The algorithm can be initialized with  $\hat{\mathbf{R}}_k^{-1}(0) = \delta \mathbf{I}$  and  $\hat{\mathbf{I}}_k^{-1}(0) = \delta \mathbf{I}$  where  $\delta$  is a scalar to ensure numerical stability. Once  $\mathbf{R}_k^{-1}(i)$  is updated, we employ another recursion to estimate  $(\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k)^{-1}$  as described by:

$$\mathbf{\Gamma}_k^{-1}(i) = \frac{\mathbf{\Gamma}_k^{-1}(i-1)}{1 - \alpha} - \frac{\mathbf{\Gamma}_k^{-1}(i-1) \gamma_k(i) \gamma_k^H(i) \mathbf{\Gamma}_k^{-1}(i-1)}{\frac{(1-\alpha)^2}{\alpha} + (1-\alpha) \gamma_k^H(i) \mathbf{\Gamma}_k^{-1}(i) \gamma_k(i)} \quad (32)$$

where  $\mathbf{\Gamma}_k(i)$  is an estimate of  $(\mathbf{C}_k^H \mathbf{R}_k^{-1}(i) \mathbf{C}_k)$  and  $\gamma_k(i) = \mathbf{C}_k^H \mathbf{r}(i) z_k(i)$ . The RLS channel estimation procedure described in [27] with  $\mathbf{\Gamma}_k$  in lieu of  $\mathbf{\Theta}_k$  is employed for estimating  $\mathbf{h}_k$ , saving computational resources and resulting in no performance loss for channel estimation. Finally, we construct the DF-CCM receiver as described by:

$$\hat{\mathbf{w}}_k(i) = \hat{\mathbf{R}}_k^{-1}(i) \left[ \hat{\mathbf{d}}_k(i) + \hat{\mathbf{T}}_k(i) \hat{\mathbf{f}}_k(i) - \mathbf{C}_k \hat{\mathbf{I}}_k^{-1}(i) \times \left( \mathbf{C}_k^H \hat{\mathbf{R}}_k^{-1}(i) \hat{\mathbf{T}}_k(i) \hat{\mathbf{f}}_k(i) + \mathbf{C}_k^H \hat{\mathbf{R}}_k^{-1}(i) \hat{\mathbf{d}}_k(i) - \nu \hat{\mathbf{h}}_k(i) \right) \right] \quad (33)$$

$$\hat{\mathbf{f}}_k(i) = \mathbf{I}_k^{-1}(i) \left[ \hat{\mathbf{T}}_k^H(i) \hat{\mathbf{w}}_k(i) - \hat{\mathbf{v}}_k(i) \right] \quad (34)$$

where  $\mathbf{d}_k(i)$  is estimated by  $\hat{\mathbf{d}}_k(i+1) = \alpha \hat{\mathbf{d}}_k(i) + (1 - \alpha) z_k^*(i) \mathbf{r}(i)$ ,  $\hat{\mathbf{T}}_k(i+1) = \alpha \hat{\mathbf{T}}_k(i) + (1 - \alpha) \hat{\mathbf{b}}_k(i) \mathbf{r}^H(i) |z_k(i)|^2$  and  $\hat{\mathbf{v}}_k(i+1) = \alpha \hat{\mathbf{v}}_k(i) + (1 - \alpha) z_k^*(i) \hat{\mathbf{b}}(i)$ . In terms of computational complexity, the CCM-RLS algorithm requires  $O(M^2)$  (feedforward section) and  $O(K^2)$  (feedback section) to suppress MAI and ISI and  $O(L_p^2)$  to estimate the channel, against  $O(M^3)$ ,  $O(K^3)$  and  $O(L_p^3)$  required by (11), (12) and (16), respectively.

### E. Constrained Minimum Variance RLS (CMV-RLS) Algorithm

Similarly to the CCM-RLS, the expressions for the DF-CMV receiver given in (14) and (15) are employed, and the matrices  $\mathbf{R}^{-1}(i)$ ,  $\mathbf{B}^{-1}(i)$  and  $(\mathbf{C}_k^T \mathbf{R}^{-1}(i) \mathbf{C}_k)^{-1}$  are recursively estimated with the aid of the matrix inversion lemma in order to reduce the computational complexity as given by :

$$\mathbf{G}(i) = \frac{\alpha^{-1} \hat{\mathbf{R}}^{-1}(i-1) \mathbf{r}(i)}{1 + \alpha^{-1} \mathbf{r}^H(i) \hat{\mathbf{R}}^{-1}(i-1) \mathbf{r}(i)} \quad (35)$$

$$\hat{\mathbf{R}}^{-1}(i) = \alpha^{-1} \hat{\mathbf{R}}^{-1}(i-1) - \alpha^{-1} \mathbf{G}(i) \mathbf{r}^T(i) \hat{\mathbf{R}}^{-1}(i-1) \quad (36)$$

and

$$\mathbf{Q}(i) = \frac{\alpha^{-1} \hat{\mathbf{B}}^{-1}(i-1) \hat{\mathbf{b}}(i)}{1 + \alpha^{-1} \hat{\mathbf{b}}^H(i) \hat{\mathbf{B}}^{-1}(i-1) \hat{\mathbf{b}}(i)} \quad (37)$$

$$\hat{\mathbf{B}}^{-1}(i) = \alpha^{-1} \hat{\mathbf{B}}^{-1}(i-1) - \alpha^{-1} \mathbf{Q}(i) \hat{\mathbf{b}}^H(i) \hat{\mathbf{B}}^{-1}(i-1) \quad (38)$$

where  $0 < \alpha < 1$  is the forgetting factor. The algorithm can be initialized with  $\hat{\mathbf{R}}^{-1}(0) = \delta \mathbf{I}$  and  $\hat{\mathbf{B}}^{-1}(0) = \delta \mathbf{I}$  where  $\delta$  is a positive constant. Once  $\hat{\mathbf{R}}^{-1}(i)$  is updated, we employ another recursion to estimate  $(\mathbf{C}_k^H \hat{\mathbf{R}}^{-1}(i) \mathbf{C}_k)^{-1}$  as described by:

$$\mathbf{\Theta}_k^{-1}(i) = \left[ \frac{\mathbf{\Theta}_k^{-1}(i-1)}{1 - \alpha} - \frac{\mathbf{\Theta}_k^{-1}(i-1) \boldsymbol{\theta}_k(i) \boldsymbol{\theta}_k^H(i) \mathbf{\Theta}_k^{-1}(i-1)}{\frac{(1-\alpha)^2}{\alpha} + (1-\alpha) \boldsymbol{\theta}_k^H(i) \mathbf{\Theta}_k^{-1}(i) \boldsymbol{\theta}_k(i)} \right] \quad (39)$$

where  $\mathbf{\Theta}_k(i)$  is an estimate of  $(\mathbf{C}_k^H \mathbf{R}^{-1}(i) \mathbf{C}_k)$  and  $\boldsymbol{\theta}_k(i) = \mathbf{C}_k^H \mathbf{r}(i)$ . For estimating the channel  $\mathbf{h}_k$ , the RLS algorithm described in [27] is employed. Finally, we construct the DF-CMV receiver as given by:

$$\hat{\mathbf{w}}_k(i) = \mathbf{R}^{-1}(i) \left[ \hat{\mathbf{T}}(i) \hat{\mathbf{f}}_k(i) - \mathbf{C}_k \mathbf{\Theta}_k^{-1}(i) \times \left( \mathbf{C}_k^H \hat{\mathbf{R}}^{-1}(i) \hat{\mathbf{T}}(i) \hat{\mathbf{f}}_k(i) - \hat{\mathbf{h}}_k(i) \right) \right] \quad (40)$$

$$\hat{\mathbf{f}}_k(i) = \hat{\mathbf{B}}^{-1}(i) \left[ \hat{\mathbf{T}}^H(i) \hat{\mathbf{w}}_k(i) \right] \quad (41)$$

where  $\hat{\mathbf{T}}(i+1) = \alpha \hat{\mathbf{T}}(i) + (1-\alpha) \hat{\mathbf{b}}_k(i) \mathbf{r}^H(i)$ . It should be remarked that the approximation on  $\hat{\mathbf{B}}$ , that is  $\mathbf{I} \approx E[\hat{\mathbf{b}}\hat{\mathbf{b}}^H]$ , can be used when the error rate is low in order to avoid the matrix computations in (37) and (38). Otherwise, in the case of moderate to high error rate, it is preferable to employ (37) and (38) in order to guarantee adequate performance of the algorithm.

## VI. SIMULATIONS

In this section, we evaluate the performance of the iterative arbitrated DF structures introduced in Section IV and the blind adaptive algorithms presented in Section V. Due to the extreme difficulty of theoretically analyzing such scheme, we adopt a simulations approach and conduct several experiments in order to verify the effectiveness of the proposed techniques. In particular, we have carried out experiments under stationary and non-stationary scenarios to assess convergence performance in terms of the the bit error rate (BER) of the proposed structure and algorithms and compared them with other recently reported algorithms and structures. Moreover, BER performance of the receivers employing the analyzed techniques is assessed for different loads, channel paths ( $L_p$ ) and profiles, and fading rates. The DS-CDMA system employs Gold sequences of length  $N = 31$ .

Because we focus on uplink scenarios, users experiment different channels. All channels assume that  $L_p = 6$  as an upper bound. We use three-path channels with relative powers  $p_{k,l}$  given by 0, -3 and -6 dB, where in each run and for each user the second path delay ( $\tau_2$ ) is given by a discrete uniform random variable (r. v.) between 1 and 4 chips and the third path delay is taken from a discrete uniform r. v. between 1 and  $5-\tau_2$  chips. It is also assumed here that the channels experienced by different users are statistically independent and identically distributed. The sequence of channel coefficients for each user  $k$  ( $k = 1, \dots, K$ ) is  $h_{k,l}(i) = p_{k,l} \alpha_{k,l}(i)$  ( $l = 0, 1, 2, \dots$ ), where  $\alpha_{k,l}(i)$ , is a complex Gaussian random sequence obtained by passing complex white Gaussian noise through a filter with approximate transfer function  $c/\sqrt{1-(f/f_d)^2}$  where  $c$  is a normalization constant,  $f_d = v/\lambda$  is the maximum Doppler shift,  $\lambda$  is the wavelength of the carrier frequency, and  $v$  is the speed of the mobile [30]. This procedure corresponds to the generation of independent sequences of correlated unit power complex Gaussian random variables ( $E[|\alpha_{k,l}^2(i)|] = 1$ ) with the path weights  $p_{k,l}$  normalized so that  $\sum_{l=1}^{L_p} p_{k,l}^2 = 1$ . The phase ambiguity derived from the blind channel estimation method in [27] is eliminated in our simulations by using the phase of  $\mathbf{g}(0)$  as a reference to remove the ambiguity and for fading channels we assume ideal phase tracking and express the results in terms of the normalized Doppler frequency  $f_d T$  (cycles/symbol). Alternatively, differential modulation can be used to account for the phase rotations.

In the following experiments, it is indicated the type of adaptive algorithms used (SG or RLS), the design criterion (CCM or CMV) and the structure (linear (L) or decision feedback (DF)). For linear receivers (L) and their algorithms

we make  $\mathbf{f}_k(i) = \mathbf{0}$  and  $\mu_f = 0$ . Amongst the analyzed DF structures, we consider:

- S-DF: the successive DF detector of [8], [9].
- P-DF: the parallel DF detector of [10], [11].
- ISS-DF: the iterative system of Woodward *et al.* [11] with S-DF in the first and second stages.
- ISP-DF: the iterative system of Woodward *et al.* [11] with S-DF in the first stage and P-DF in the second stage.
- SPA-DF: the proposed successive parallel arbitrated receiver.
- ISPAS-DF: the proposed iterative detector with the novel SPA-DF in the first stage and the S-DF in the second stage.
- ISPAP-DF: the proposed iterative receiver with the SPA-DF in the first stage and the P-DF in the second stage.

For the CCM based algorithms, we employ  $\nu = 1$  in order to ensure convexity. The experiments are averaged over 200 experiments and the parameters of the algorithms are optimized for each scenario. We stress that the results are shown in Figs. 4 to 8 in terms of the average BER [1] and average BER amongst the  $K$  users in the system, except for Figs. 9 and 10, where the individual BER performance of each user is shown.

### A. BER Convergence Performance

In what follows, we assess the average BER convergence performance of the analyzed adaptive DF receiver techniques and algorithms. The BER convergence performance of the receivers is shown for SG and RLS algorithms, in Figs. 4 and 5, respectively. We consider a non-stationary scenario, where the system starts with  $K = 8$  users and at time  $i = 800$ , 4 additional users enter the system, totalling  $K = 12$  users, and the blind adaptive algorithms are subject to new interferers/users in the environment. For the sake of comparison, we also include the curves for supervised NLMS and RLS [29] adaptive algorithms, which are trained with 200 symbols provided by a pilot channel (at  $i = 1, \dots, 200$  and  $i = 801, \dots, 1000$ ) and then switch to decision-directed mode. It is assumed that the system has an ideal power control and signals of the different users reach the base station with the same average  $E_b/N_0$ . Note that given the performance of current power control algorithms, ideal power control is not far from a realistic situation.

The algorithms for DF receivers are initialized with a feedforward filter  $\mathbf{w}_k$  equal to the signature sequence and a feedback filter  $\mathbf{f}_k$  with zeros and they gradually adapt in order to cancel the interference. Note that they do not lock to an undesired user because of the blind channel estimation that allows the receiver to use the effective signature sequence. The results indicate that the CCM design criterion is superior to the CMV approach, for both SG and RLS algorithms. Another conclusion from the curves in Figs. 4 and 5 is that CCM-based blind algorithms achieve a performance very close to the trained algorithms, leading to significant savings in spectral efficiency. Regarding the structures of the receivers, we note that DF receivers are significantly better than linear detectors. In fact, we attack the problem of the receivers presented in

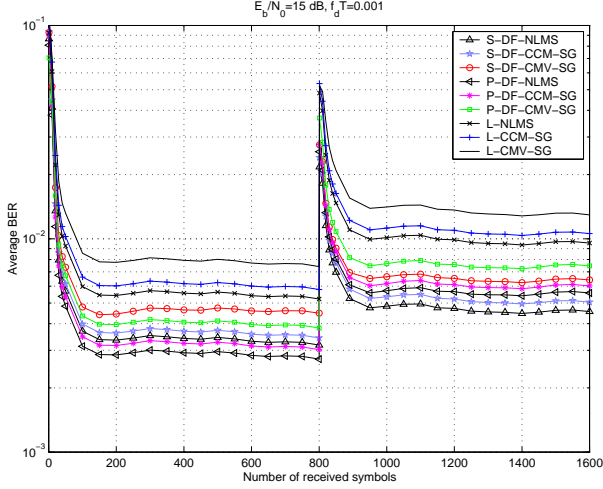


Fig. 4. BER convergence performance of SG algorithms.

[15], [18] that operate well in lightly loaded systems, but do not perform well in moderate and heavily loaded situations, by cancelling the interferers with the DF section. In particular, P-DF schemes outperform S-DF in low BER situations, whereas for moderate to higher BER levels S-DF systems are less affected by error propagation.

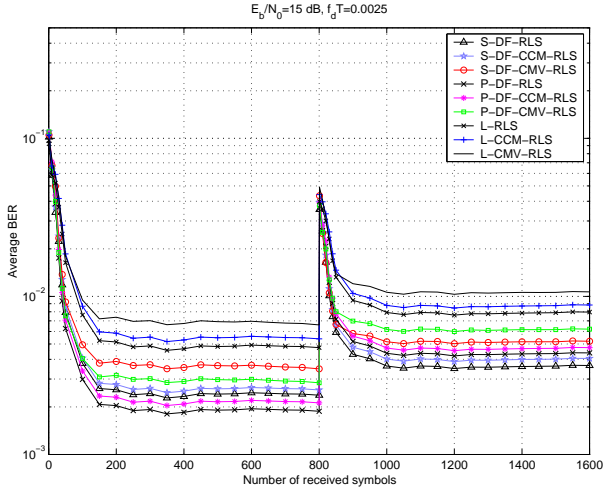


Fig. 5. BER convergence performance of RLS algorithms.

Another important conclusion from our studies is that RLS algorithms can deal with faster fading rates and effectively accommodate more users in the system at the cost of a quadratic complexity, whereas SG techniques cannot deal with large systems or very high load ( $K/N$  close to 1). Because the scenario in the experiments assumed ideal power control, the SG algorithms present a good convergence performance although for scenarios without power control (near-far situations) the performance of these algorithms is subject to the eigenvalue spread of the covariance matrix of the received vector  $\mathbf{r}(i)$ . Specifically, when the eigenvalue spread of the covariance matrix of the received vector  $\mathbf{r}(i)$  is large SG algorithms perform poorly, whereas the rate of convergence

of RLS algorithms is invariant to such situation [29]. Hence, for large systems or those that do not have good power control RLS recursions are the most appropriate.

Let us now consider the proposed SPA-DF and the combined iterative DF system, namely ISPAP-DF and ISPAS-DF. Simulation experiments with RLS algorithms were conducted to determine how many arbitrated branches should be used and to account for the impact of additional branches upon performance. We designed the novel DF receivers with  $L = 2, 4, 8$  parallel branches and compared their BER performance with the existing ISS-DF and ISP-DF structures, as depicted in Fig. 6. The results show that the novel SPA-DF, ISPAP-DF and ISPAS-DF significantly outperform the ISS-DF and ISP-DF structures, and their performances improve as the number of parallel branches increase. In this regard, we also notice that the gains of performance obtained through additional branches decrease as additional branches are added, resulting in marginal improvements for more than  $L = 4$  branches. For this reason, we adopt  $L = 4$  for the remaining experiments because it presents a very attractive trade-off between performance and complexity. Another conclusion from the curves in Fig. 6 is that the proposed SPA-DF, ISPAP-DF and ISPAS-DF receiver techniques obtain substantial gains in performance over existing iterative DF techniques, namely, the ISP-DF and the ISS-DF of [11].

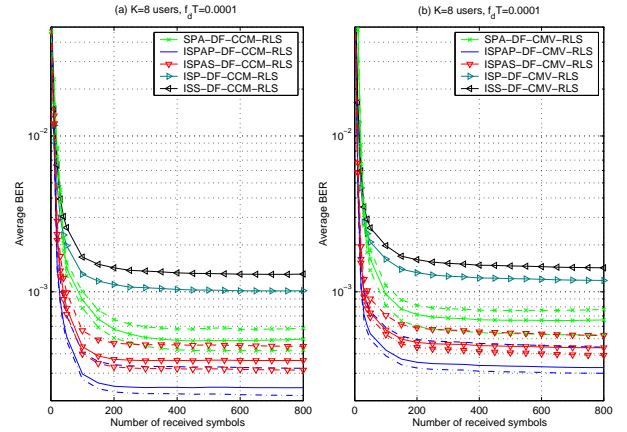


Fig. 6. BER convergence performance of RLS algorithms with iterative receivers for different numbers of parallel branches  $L$  at  $E_b/N_0 = 15$  dB in a slow fading environment (a) CCM (L=2 - dashed line, L=4 - solid line and L=8 - dash-dotted line).

### B. BER Performance versus $E_b/N_0$ , $K$ and User Index

In this part, the BER performance of the different receiver techniques is further investigated and the receivers process 2000 symbols to obtain the curves. In particular, the average BER performance of the receivers versus  $E_b/N_0$  and number of users ( $K$ ) is depicted in Figs. 7 and 8, whereas the individual BER performance versus the user indices is shown in Figs. 9 and 10.

A comparison of the CMV and the CCM design criteria with RLS algorithms is carried out in experiments whose results are shown in Figs. 7 and 8. The curves reveal that DF detectors

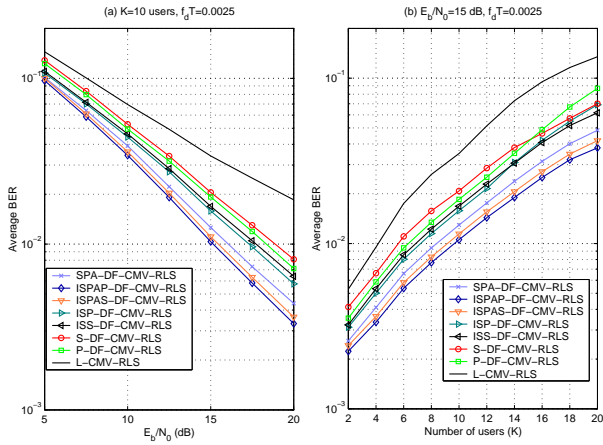


Fig. 7. Performance of CMV-RLS algorithms in a dynamic environment in terms of BER versus (a)  $E_b/N_0$  with  $K = 10$  users and (b) number of users ( $K$ ) at  $E_b/N_0 = 15$  dB.

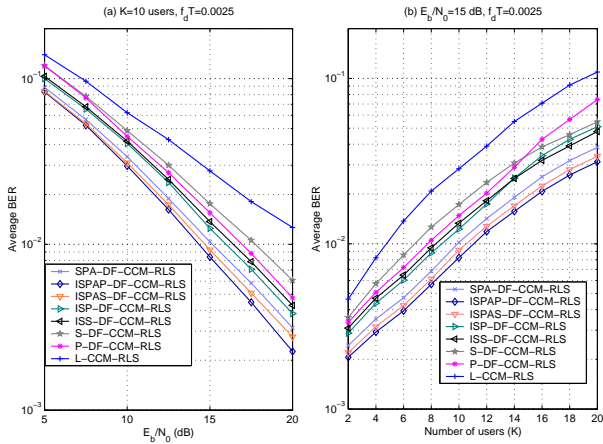


Fig. 8. Performance of CCM-RLS algorithms in a dynamic environment in terms of BER versus (a)  $E_b/N_0$  with  $K = 10$  users and (b) number of users ( $K$ ) at  $E_b/N_0 = 15$  dB.

are significantly superior to linear receivers and that the CCM-RLS algorithm outperforms the CMV-RLS techniques in all situations. With respect to the performance, the best results are obtained with the ISPAP-DF receiver structure, that can save up to 2.5 dB for the same BER as compared to the iterative receivers of [11] (ISP-DF and ISS-DF). In comparison with linear receivers, the proposed ISPAP-DF system obtains savings of up to 7 dB for the BER performance. In general, the curves in Figs. 7 and 8 reveal that the novel iterative arbitrated DF schemes, namely the SPA-DF, ISPAP-DF and ISPAS-DF, can offer considerable gains as compared to existing DF and linear receivers and support systems with higher loads, through mitigation of the effects of error propagation.

The last two scenarios, shown in Figs. 9 and 10, consider the individual BER performance of the users. From the results, we observe that a disadvantage of S-DF relative to P-DF is that it does not provide uniform performance over the user population. We also notice that for the S-DF receivers, user 1 achieves the same performance of their linear receivers counterparts, and as the successive cancellation is performed

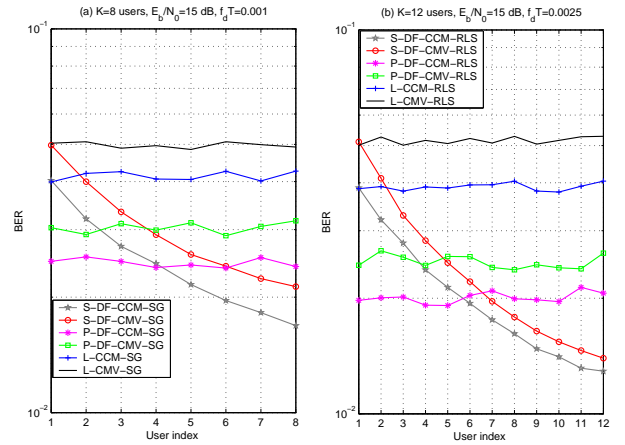


Fig. 9. Performance of the receivers in a fading environment in terms of individual BER versus user index for (a) SG and (b) RLS algorithms.

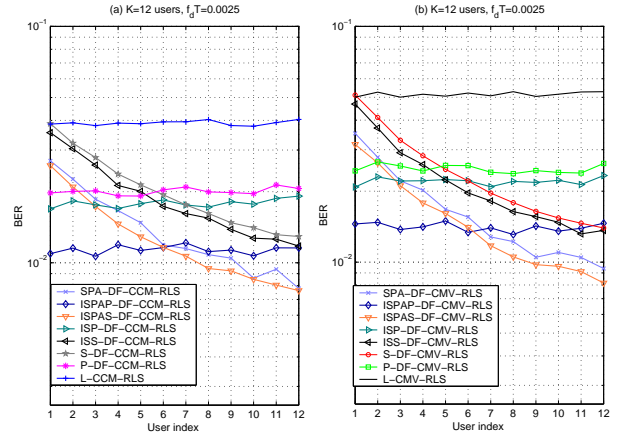


Fig. 10. Performance of the receivers in terms of individual BER versus user index for (a) CCM-RLS and (b) CMV-RLS algorithms.

users with higher indices benefit from the interference cancellation. The same non-uniform performance is verified for the proposed SPA-DF, the existing ISS-DF and the novel ISPAS-DF. Conversely, the new ISPAP-DF, the existing P-DF and the existing ISP-DF provide uniform performance over the users which is an important goal for the uplink of DS-CDMA systems. In particular, the novel ISPAP-DF detector achieves the best performance of the analyzed structures and is significantly superior to the ISP-DF and to the P-DF, that suffers from error propagation.

## VII. CONCLUSIONS

Blind adaptive SG and RLS type algorithms based on the CMV and CCM performance criteria were developed for estimating the parameters of DF receivers in uplink scenarios with multipath. The CCM-based blind algorithms have shown a performance that is very close to that of trained algorithms without the need for pilot channels. A novel SPA-DF structure was presented and combined with iterative techniques for use with cascaded DF stages, resulting in new iterative DF schemes, the ISPAS-DF and the ISPAP-DF, that can offer

substantial gains in performance over existing linear and DF detectors and mitigate more effectively the deleterious effects of error propagation. In particular, the proposed ISPAP-DF structure has achieved the best performance amongst all analyzed receivers and is able to provide uniform performance over the user population.

## APPENDIX

In what follows, an analysis of the CCM method and its convergence properties is carried out for the linear receiver case ( $\mathbf{f}_k = \mathbf{0}$ ), extending previous results on its convexity for both complex and multipath signals. We believe that it provides a good starting point (better than the CMV design) for performing the cancellation of the associated users by the feedforward section of the DF-CCM receiver.

Let us express the cost function  $J_{CM} = E[(|\mathbf{w}_k^H \mathbf{r}|^2 - 1)^2]$  as  $J_{CM} = (E[|z_k|^4] - 2E[|z_k|^2] + 1)$ , drop the time index (i) for simplicity, assume a stationary scenario and that  $b_k$ ,  $k=1, \dots, K$  are statistically independent i.i.d complex random variables with zero mean and unit variance,  $b_k$  and  $\mathbf{n}$  are statistically independent. Let us also define  $\mathbf{x} = \sum_{k=1}^K A_k b_k \tilde{\mathbf{s}}_k$ ,  $\mathbf{C}_k \mathbf{h}_k = \tilde{\mathbf{s}}_k$ ,  $\mathbf{Q} = E[\mathbf{x}\mathbf{x}^H]$ ,  $\mathbf{P} = E[\boldsymbol{\eta}\boldsymbol{\eta}^H]$ ,  $\mathbf{R} = \mathbf{Q} + \mathbf{P} + \sigma^2 \mathbf{I}$  and alternatively express the received vector by  $\mathbf{r}(i) = \mathbf{x}(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i)$ , where  $\boldsymbol{\eta}(i)$  is the ISI. Considering user 1 as the desired one we let  $\mathbf{w}_1 = \mathbf{w}$  and define  $u_k = A_k^* \tilde{\mathbf{s}}_k^H \mathbf{w}$ ,  $\mathbf{u} = \mathbf{A}^H \tilde{\mathbf{S}}^H \mathbf{w} = [u_1 \dots u_K]^T$ , where  $\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}_1 \dots \tilde{\mathbf{s}}_K]$ ,  $\mathbf{A} = \text{diag}(A_1 \dots A_K)$  and  $\mathbf{b} = [b_1 \dots b_K]^T$ . Using the constraint  $\mathbf{C}_1^H \mathbf{w} = \nu \hat{\mathbf{h}}_1$  we have for the desired user the condition  $u_1 = (A_1^* \tilde{\mathbf{s}}_1^H) \mathbf{w} = A_1^* \mathbf{h}_1^H \mathbf{w} = \nu A_1^* \hat{\mathbf{h}}_1^H \hat{\mathbf{h}}_1$ . In the absence of noise and neglecting ISI, the (user 1) cost function can be expressed as  $J_{CM}(\mathbf{w}) = E[(\mathbf{u}^H \mathbf{b} \mathbf{b}^H \mathbf{u})^2] - 2E[(\mathbf{u}^H \mathbf{b} \mathbf{b}^H \mathbf{u})] + 1 = 8(\sum_{k=1}^K u_k u_k^*)^2 - 4 \sum_{k=1}^K (u_k u_k^*)^2 - 4 \sum_{k=1}^K u_k u_k^* + 1 = 8(D + \sum_{k=2}^K u_k u_k^*)^2 - 4D^2 - 4 \sum_{k=2}^K (u_k u_k^*)^2 - 4D - 4 \sum_{k=2}^K (u_k u_k^*) + 1$ , where  $D = u_1 u_1^* = \nu^2 |A_1|^2 |\hat{\mathbf{h}}_1^H \hat{\mathbf{h}}_1|^2$ . To examine the convergence properties of the optimization problem in (10), we proceed similarly to [16]. Under the constraint  $\mathbf{C}_1^H \mathbf{w} = \nu \hat{\mathbf{h}}_1$ , we have:

$$J_{CM}(\mathbf{w}) = \tilde{J}_{CM}(\bar{\mathbf{u}}) = 8(D + \bar{\mathbf{u}}^H \bar{\mathbf{u}})^2 - 4D^2 - 4 \sum_{k=2}^K (u_k u_k^*)^2 - 4D - 4(\bar{\mathbf{u}}^H \bar{\mathbf{u}}) + 1 \quad (42)$$

where  $\bar{\mathbf{u}} = [u_2, \dots, u_K]^T = \mathbf{B}\mathbf{w}$ ,  $\mathbf{B} = \mathbf{A}'^H \tilde{\mathbf{S}}'^H$ ,  $\tilde{\mathbf{S}}' = [\tilde{\mathbf{s}}_2 \dots \tilde{\mathbf{s}}_K]$  and  $\mathbf{A}' = \text{diag}(A_2 \dots A_K)$ . To evaluate the convexity of  $\tilde{J}_{CM}(\cdot)$ , we compute its Hessian ( $\mathbf{H}$ ) using the rule  $\mathbf{H} = \frac{\partial}{\partial \bar{\mathbf{u}}^H} \frac{\partial(\tilde{J}_{CM}(\bar{\mathbf{u}}))}{\partial \bar{\mathbf{u}}}$  that yields:

$$\mathbf{H} = \left[ 16(D-1/4)\mathbf{I} + 16\bar{\mathbf{u}}^H \bar{\mathbf{u}} \mathbf{I} + 16\bar{\mathbf{u}} \bar{\mathbf{u}}^H - 16 \text{diag}(|u_2|^2 \dots |u_K|^2) \right] \quad (43)$$

Specifically,  $\mathbf{H}$  is positive definite if  $\mathbf{a}^H \mathbf{H} \mathbf{a} > 0$  for all nonzero  $\mathbf{a} \in \mathbb{C}^{K-1 \times K-1}$  [29]. The second, third and fourth terms of (46) yield the positive definite matrix  $16(\bar{\mathbf{u}} \bar{\mathbf{u}}^H + \text{diag}(\sum_{k=3}^K |u_k|^2, \sum_{k=2, k \neq 3}^K |u_k|^2, \dots, \sum_{k=3, k \neq K}^K |u_k|^2))$ , whereas the first term provides the condition  $\nu^2 |A_1|^2 |\hat{\mathbf{h}}_1^H \hat{\mathbf{h}}_1|^2 \geq 1/4$  that ensures the convexity of

$\tilde{J}_{CM}(\cdot)$  in the noiseless case. Because  $\bar{\mathbf{u}} = \mathbf{B}\mathbf{w}$  is a linear function of  $\mathbf{w}$  then  $\tilde{J}_{CM}(\bar{\mathbf{u}})$  being a convex function of  $\bar{\mathbf{u}}$  implies that  $J_{CM}(\mathbf{w}) = \tilde{J}_{CM}(\mathbf{B}\mathbf{w})$  is a convex function of  $\mathbf{w}$ . Since the extrema of the cost function can be considered for small  $\sigma^2$  a slight perturbation of the noise-free case [16], the cost function is also convex for small  $\sigma^2$  when  $\nu^2 |A_1|^2 |\hat{\mathbf{h}}_1^H \hat{\mathbf{h}}_1|^2 \geq 1/4$ . If we assume ideal channel estimation ( $|\hat{\mathbf{h}}_1^H \hat{\mathbf{h}}_1| = 1$ ) and  $\nu = 1$ , our result reduces to  $|A_1|^2 \geq 1/4$ , which is the same found in [31]. For larger values of  $\sigma^2$ , we remark that the term  $\nu$  can be adjusted in order to make the cost function  $J_{CM}$  in (10) convex, as pointed out in [16].

To derive a normalized step size for the algorithm in (24) and (25) let us drop the time index (i) for simplicity and write the constant modulus cost function  $J_{CM} = (|\mathbf{w}_k^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}}|^2 - 1)^2$  as a function of (24) and (25):

$$J_{CM} = (|\mathbf{P}_k(\mathbf{w}_k - \mu_w \mathbf{r} e_k z_k^*)^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}} - \mu_f e_k^* z_k \hat{\mathbf{b}}^H \hat{\mathbf{b}} + (\mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{h}_k)^H \mathbf{r}|^2 - 1)^2 \quad (44)$$

If we substitute  $\mathbf{P}_k = \mathbf{I} - (\mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{C}_k^H)$  into the first term of (44) and use  $\mathbf{C}_k^H \mathbf{w}_k = \mathbf{h}_k$  we can simplify (44) and obtain:

$$J_{CM} = (|z_k - \mu_w e_k z_k \mathbf{r}^H \mathbf{P}_k \mathbf{r} - \mu_f e_k z_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}|^2 - 1)^2 \quad (45)$$

Next, if we take the gradient of  $J_{CM}$  with respect to  $\mu_w$  and equal it to zero, we have:

$$\nabla J_{\mu_w} = 2(|z_k - \mu_w e_k z_k \mathbf{r}^H \mathbf{P}_k \mathbf{r} - \mu_f e_k z_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}|^2 - 1) \times \frac{d}{d\mu_w} |z_k - \mu_w e_k z_k \mathbf{r}^H \mathbf{P}_k \mathbf{r} - \mu_f e_k z_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}|^2 = 0 \quad (46)$$

From the above expression it is clear that this minimization leads to four possible solutions, namely:

$$\mu_w^{n.1} = \mu_w^{n.2} = \frac{1 - \mu_f e_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}}{e_k \mathbf{r}^H \mathbf{P}_k \mathbf{r}}, \mu_w^{n.3} = \frac{(|z_k| - 1) - \mu_f |z_k| e_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}}{|z_k| e_k \mathbf{r}^H \mathbf{P}_k \mathbf{r}}, \mu_w^{n.4} = \frac{(|z_k| + 1) - \mu_f |z_k| e_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}}{|z_k| e_k \mathbf{r}^H \mathbf{P}_k \mathbf{r}} \quad (47)$$

By computing the second derivative of (44) one can verify that it is always positive for the third and fourth solutions above, indicating the minimum point. It should be remarked that the solution for  $\mu_f$  is analogous to  $\mu_w$  and leads to the same relations. Hence, we choose  $\mu_w = \frac{(|z_k|+1)-\mu_f |z_k| e_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}}{|z_k| e_k \mathbf{r}^H \mathbf{P}_k \mathbf{r}}$  and introduce again the convergence factors  $\mu_{0w}$  and  $\mu_{0f}$  so that the algorithms can operate with adequate step sizes that are usually small to ensure good performance, and thus we have  $\mu_w = \mu_{0w} \frac{(|z_k|+1)-\mu_f |z_k| e_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}}{|z_k| e_k \mathbf{r}^H \mathbf{P}_k \mathbf{r}}$  and  $\mu_f = \mu_{0f} \frac{(|z_k|+1)-\mu_w |z_k| e_k \mathbf{r}^H \mathbf{P}_k \mathbf{r}}{|z_k| e_k \hat{\mathbf{b}}^H \hat{\mathbf{b}}}$ .

To derive a normalized step size for the SG algorithm in (26) and (27) let us drop again the time index (i) for simplicity and write the minimum variance cost function  $J = |\mathbf{w}_k^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}}|^2$  as:

$$J_{MV} = |\mathbf{P}_k(\mathbf{w}_k - \mu_w \mathbf{r} x_k^*)^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}} - \mu_f x_k \hat{\mathbf{b}}^H \hat{\mathbf{b}} + \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{h}_k)^H \mathbf{r}|^2 \quad (48)$$

If we take the gradient of  $J_{MV}$  with respect to  $\mu_w$  and equal it to zero, we get:

$$\nabla J_{\mu_w} = 2|\mathbf{P}_k(\mathbf{w}_k - \mu_w \mathbf{r} x_k^*)^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}} - \mu_f x_k \hat{\mathbf{b}}^H \hat{\mathbf{b}} + \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{h}_k)^H \mathbf{r}| \times (-\mathbf{P}_k \mathbf{r} x_k^*)^H \mathbf{r} = 0 \quad (49)$$

If we substitute  $\mathbf{P}_k = \mathbf{I} - (\mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{C}_k^H)$  into the first term of (49) and use  $\mathbf{C}_k \mathbf{w}_k = \mathbf{h}_k$  we can eliminate the third term of (49) and obtain the solution:

$$\mu_w = \frac{x_k(1 - \mu_f \hat{\mathbf{b}}^H \hat{\mathbf{b}})}{x_k(\mathbf{r}^H \mathbf{P}_k \mathbf{r})} = \frac{(1 - \mu_f \hat{\mathbf{b}}^H \hat{\mathbf{b}})}{\mathbf{r}^H \mathbf{P}_k \mathbf{r}} \quad (50)$$

Note that we introduce again a convergence factor  $\mu_{0_w}$  so that the algorithm can operate with adequate step sizes that are usually small to ensure good performance, and thus we have  $\mu_w = \mu_{0_w} \frac{(1 - \mu_f \hat{\mathbf{b}}^H \hat{\mathbf{b}})}{\mathbf{r}^H \mathbf{P}_k \mathbf{r}}$ . Next, we take the gradient of  $J_{MV}$  with respect to  $\mu_f$  and equal it to zero:

$$\nabla J_{\mu_f} = 2|\mathbf{P}_k(\mathbf{w}_k - \mu_w \mathbf{r} x_k^*)^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}} - \mu_f x_k \hat{\mathbf{b}}^H \hat{\mathbf{b}} + \mathbf{C}_k(\mathbf{C}_k^H \mathbf{C}_k)^{-1} \mathbf{h}_k)^H \mathbf{r}| \times (-x_k \hat{\mathbf{b}}^H \hat{\mathbf{b}})^H \mathbf{r} = 0 \quad (51)$$

where it is noticed that the conditions are the same as for  $\mu_w$ . Thus we proceed similarly to obtain the step size  $\mu_f$ , which is given by  $\mu_f = \frac{(1 - \mu_w \mathbf{r}^H \mathbf{P}_k \mathbf{r})}{\hat{\mathbf{b}}^H \hat{\mathbf{b}}}$ . Remark again that we a convergence factor  $\mu_{0_f}$  is applied so that the algorithm can operate with adequate step sizes that are usually small to ensure good performance, and thus we employ  $\mu_f = \mu_{0_f} \frac{(1 - \mu_w \mathbf{r}^H \mathbf{P}_k \mathbf{r})}{\hat{\mathbf{b}}^H \hat{\mathbf{b}}}$ .

Here, we discuss the suitability of the matrix  $\mathbf{R}_k$ , that arises from the CCM method, for use in channel estimation. From the analysis in Appendix I for the linear receiver, we have for an ideal and asymptotic case that  $u_k = (\mathbf{A}_1^H \mathbf{s}_k^H) \mathbf{w}_1 \approx 0$ , for  $k = 2, \dots, K$ . Then,  $\mathbf{w}_1^H \mathbf{r} \approx \mathbf{A}_1 \mathbf{b}_1 \mathbf{w}_1^H \mathbf{s}_1 + \mathbf{w}_1^H \mathbf{n}$  and  $|\mathbf{w}_1^H \mathbf{r}|^2 \approx \mathbf{A}_1^2 |\mathbf{w}_1^H \mathbf{s}_1|^2 + \mathbf{A}_1 \mathbf{b}_1 (\mathbf{w}_1 \mathbf{s}_1) \mathbf{n}^H \mathbf{w}_1 + \mathbf{A}_1 \mathbf{b}_1^* (\mathbf{s}_1^H \mathbf{w}_1) \mathbf{w}_1^H \mathbf{n} + \mathbf{w}_1^H \mathbf{n} \mathbf{n}^H \mathbf{w}_1$ . Therefore, we have for the desired user (i.e. user 1):

$$\begin{aligned} \mathbf{R}_1 &= E[|\mathbf{w}_1^H \mathbf{r}|^2 \mathbf{r} \mathbf{r}^H] \\ &\cong \mathbf{A}_1^2 |\mathbf{w}_1^H \mathbf{s}_1|^2 \mathbf{R} + \mathbf{A}_1 \mathbf{w}_1^H \mathbf{s}_1 E[b_1 \mathbf{n}^H \mathbf{w}_1 \mathbf{r} \mathbf{r}^H] + \\ &\quad \mathbf{A}_1 \mathbf{s}_1^H \mathbf{w}_1 E[b_1^* \mathbf{w}_1^H \mathbf{n} \mathbf{r} \mathbf{r}^H] + E[\mathbf{w}_1^H \mathbf{n} \mathbf{n}^H \mathbf{w}_1 \mathbf{n} \mathbf{n}^H] + \sigma^2 \mathbf{Q} \mathbf{w}_1^H \mathbf{w}_1 \\ &\cong \mathbf{A}_1^2 |\mathbf{w}_1^H \mathbf{s}_1|^2 \mathbf{R} + \mathbf{A}_1 \mathbf{w}_1^H \mathbf{s}_1 E[b_1 \mathbf{n}^H \mathbf{w}_1 \mathbf{r} \mathbf{r}^H] + E[|\mathbf{w}_1^H \mathbf{n}|^2 \mathbf{n} \mathbf{n}^H] \\ &\quad + \mathbf{A}_1 \mathbf{s}_1^H \mathbf{w}_1 E[b_1^* \mathbf{w}_1^H \mathbf{n} \mathbf{r} \mathbf{r}^H] + \sigma^2 (\mathbf{R} - \sigma^2 \mathbf{I}) \mathbf{w}_1^H \mathbf{w}_1 \\ &\cong (\mathbf{A}_1^2 |\mathbf{w}_1^H \mathbf{s}_1|^2 + \sigma^2) \mathbf{R} + \mathbf{A}_1^2 \sigma^2 (\mathbf{w}_1^H \mathbf{s}_1) (\mathbf{w}_1 \mathbf{s}_1^H) \\ &\quad + \mathbf{A}_1^2 \sigma^2 (\mathbf{s}_1^H \mathbf{w}_1) (\mathbf{s}_1 \mathbf{w}_1^H) + \sigma^4 [\text{diag}(|w_1|^2, \dots, |w_N|^2) \\ &\quad + \mathbf{w}_1 \mathbf{w}_1^H] - \sigma^4 \mathbf{w}_1^H \mathbf{w}_1 \mathbf{I} \\ &\cong \mathbf{A}_1^4 \left[ \left( \frac{|\mathbf{w}_1^H \mathbf{s}_1|^2}{\mathbf{A}_1^2} + \frac{\sigma^2}{\mathbf{A}_1^2} \right) \mathbf{R} + \frac{\sigma^2}{\mathbf{A}_1^2} ((\mathbf{w}_1^H \mathbf{s}_1) (\mathbf{w}_1 \mathbf{s}_1^H) \right. \\ &\quad \left. + \sigma^2 (\mathbf{s}_1^H \mathbf{w}_1) (\mathbf{s}_1 \mathbf{w}_1^H)) \right] + \frac{\sigma^4}{\mathbf{A}_1^4} ([\text{diag}(|w_1|^2, \dots, |w_N|^2) \\ &\quad \left. + \mathbf{w}_1 \mathbf{w}_1^H] - \mathbf{w}_1^H \mathbf{w}_1 \mathbf{I}) \right] \\ &\cong \alpha \mathbf{R} + \tilde{\mathbf{N}} \end{aligned} \quad (52)$$

where  $\mathbf{R} = \mathbf{Q} + \sigma^2 \mathbf{I}$ ,  $\mathbf{Q} = E[\mathbf{x} \mathbf{x}^H] = \sum_{k=1}^K |A_k|^2 \mathbf{s}_k \mathbf{s}_k^H$ . From (52), it can be seen that  $\mathbf{R}_k$  can be approximated by

$\mathbf{R}$  multiplied by a scalar factor  $\alpha$  plus a noise-like term  $\tilde{\mathbf{N}}$ , that for sufficient  $E_b/N_0$  has an insignificant contribution. In addition, when the symbol estimates  $z_k = \mathbf{w}_k^H \mathbf{r}$  are reliable, that is the cost function in (10) is small ( $J_{CM} \ll 1$ ), then  $|z_k|^2$  has small variations around unity for both linear and DF detectors (note that  $z_k = \mathbf{w}_k^H \mathbf{r} - \mathbf{f}_k^H \hat{\mathbf{b}}$  for the DF receivers), yielding the approximation

$$E[|z_k|^2 \mathbf{r} \mathbf{r}^H] = E[\mathbf{r} \mathbf{r}^H] + E[(|z_k|^2 - 1) \mathbf{r} \mathbf{r}^H] \cong E[\mathbf{r} \mathbf{r}^H] = \mathbf{R} \quad (53)$$

Therefore, we conclude that the channel estimation can be performed on  $\mathbf{R}_k$  in lieu of  $\mathbf{R}$ , since the properties of the matrix  $\mathbf{R}$  studied in [26], [27] hold for  $\mathbf{R}_k$ .

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